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it represents the development of the binomial with negative exponents, *i.e.*, $(1-x)^{-m}$, into a finite series with a remainder term. The development is therefore good whether the series is finite, convergent or divergent.

If considered in the order:

$$\frac{1 - \sum_{t=0}^{t=m-1} c_{n+t} (1-x)^t x^{n+1}}{(1-x)^m} = \sum_{t=0}^{t=n} c_{m-1+t, m-1} x^t,$$

it presents an extension of the theorem, that the difference of the same powers is divisible by the difference of the first powers.

Let, for illustration, $n = 3$, $m = 1, 2, 3, \dots$, and we shall have successively:

$$\frac{1-x^4}{1-x} = 1 + x + x^2 + x^3,$$

$$\frac{1-x^4-4(1-x)x^4}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3,$$

$$\frac{1-x^4-4(1-x)x^4-10(1-x)^2x^4}{(1-x)^3} = 1 + 3x + 6x^2 + 10x^3,$$

etc.; where the ubiquitous binomial coefficients are recognized in their usual rôle.

IV. AN ERRONEOUS RULE FOR FINDING A HYPOTENUSE, WITH A COROLLARY.

By M. W. JACOBS, JR., Harrisburg, Pa.

To find the hypotenuse of a right-angled triangle, take half the length of the longer leg and add the length of the shorter leg. This was the rule nearly all the boys of an arithmetic class were found to be using. Much to the instructor's surprise, the correct answer was thus obtained to almost all of the examples. An inspection of the problems correctly solved in this manner showed that the ratio of the longer to the shorter leg was 4 : 3.

That this is the only ratio for which the boys' rule holds true may be demonstrated as follows: Let a = longer leg; b = shorter leg; h = hypotenuse. By the boys' rule

$$h = \frac{1}{2}a + b = \frac{a + 2b}{2}.$$

Then

$$h^2 = \frac{a^2 + 4ab + 4b^2}{4}.$$

But $h^2 = a^2 + b^2$. Hence

$$a^2 + b^2 = \frac{a^2 + 4ab + 4b^2}{4}.$$

Then $4a^2 + 4b^2 = a^2 + 4ab + 4b^2$; whence $3a^2 = 4ab$; therefore $a/b = 4/3$. The

ratio a/b has the *single value* $4/3$ (when in lowest terms); the rule holds for this but for no other value.

Corollary: Generalizing by substituting the rational fraction t/u in place of $1/2$ in the boys' rule, we get:

$$h = \frac{t}{u}a + b = \frac{ta + ub}{u};$$

then

$$h^2 = a^2 + b^2 = \frac{t^2a^2 + 2tuab + u^2b^2}{u^2}.$$

Whence $u^2a^2 - t^2a^2 = 2tuab$, and $a(u^2 - t^2) = 2tub$.

Consequently

$$\frac{b}{a} = \frac{u^2 - t^2}{2tu}.$$

The two legs are therefore in the proportion: $(u^2 - t^2) : 2tu$. Let us take exactly the values $u^2 - t^2$ and $2tu$ for the sides.

Hence the hypotenuse $= \sqrt{(u^2 - t^2)^2 + (2tu)^2} = u^2 + t^2$. Therefore

$$a = u^2 - t^2; \quad b = 2tu; \quad h = u^2 + t^2, \quad (1)$$

where t and u are integers, to be chosen at pleasure, (except that $u > t$, since a is to be positive). More generally we may write

$$a = c(u^2 - t^2); \quad b = c(2tu); \quad h = c(u^2 + t^2), \quad (2)$$

which is the well-known formula for Pythagorean triangles in its most general form. The particular case, $c = 1$, gives the primitive types.

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REVIEWS.

HISTORY OF THE THEORY OF NUMBERS.

History of the Theory of Numbers. Volume I: *Divisibility and Primality.* By LEONARD EUGENE DICKSON. Carnegie Institution of Washington, 1919, publication number 256, Vol. I. 14 + 486 pages. Price, in paper, \$7.50; in cloth, \$8.00.

There are at least three great ends to serve by the history of science: to enrich the general culture and intellectual life of cultivated people; to enable a scientific worker quickly to orient himself in a chapter of science so as to proceed most readily to a detailed mastery of its literature; to enable a scientific worker to ascertain with completeness what has already been attained in a given subject. To serve any one of these three purposes requires a very different